

1.1

Natural Numbers, \mathbb{N} : The counting numbers starting at 1: $\{1, 2, 3, \dots\}$.

Whole Numbers, W : The counting numbers again, but starting at 0: $\{0, 1, 2, 3, \dots\}$.

Integers, \mathbb{Z} : The whole numbers and their opposites: $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.

Rational numbers, Q : Rational numbers are all the possible ratios of two integers, provided that 0 is not in the denominator. It is important to note that as decimals, rational numbers either stop or repeat. For example, $\frac{1}{2} = 0.5$ (stops), $\frac{1}{3} = 0.33333\dots$ (repeats), $\frac{1}{4} = 0.25$ (stops), $\frac{1}{5} = 0.2$ (stops), $\frac{1}{6} = 0.166666\dots$ (repeats), $\frac{6}{2} = 3$ (stops), $\frac{60}{4} = 15$ (stops), etc.

Irrational numbers, I : Irrational numbers are all the numbers that can't be written as a ratio of two integers. It is important to note that as decimals, irrational numbers neither stop nor repeat. For example, π is an irrational number that is approximately 3.14159. When we write 3.14159, we are actually writing a rational number approximation for the irrational number π . Another example of an irrational number is the square root of 2. The square root of 2 does not stop and it does not have a repeating part. It is approximately 1.4142, but again 1.4142 is a rational number approximation.

Real numbers, \mathbb{R} : The real numbers are all the rational numbers and all the irrational numbers.

Prime number: A prime is a natural number, greater than 1, that has only 2 factors. The factors of a prime number are 1 and itself. The only way to get a prime number by multiplication is taking 1 times the prime number. $\{2, 3, 5, 7, 11, 13, 17, 19, \dots\}$

Composite number: A composite number is a natural number, greater than 1, that is not prime. Composite numbers have more than 2 factors. $\{4, 6, 8, 9, 10, 12, 14, 15, 16, 18, \dots\}$

Which are prime and which are composite?

39?

48?

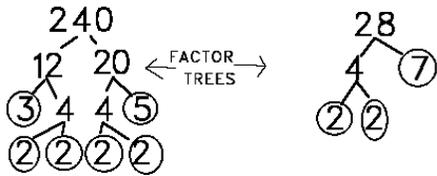
57?

59?

0?

We might use a factor tree:

... or repeated division by prime factors:



$$240 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$$

$$240 = 2^4 \cdot 3 \cdot 5$$

$$28 = 2 \cdot 2 \cdot 7$$

$$28 = 2^2 \cdot 7$$

$$\overline{)168}$$

When we write a number as a product of primes, we call it the **prime factorization** of the number.

$$\frac{\text{numerator}}{\text{denominator}} = \text{numerator} \div \text{denominator}$$

Reduce: $\frac{16}{20}$

Ex. $\frac{5}{9} \cdot \frac{10}{7}$

Ex. $\frac{7}{6} \div \frac{9}{10}$

Ex. $2\frac{1}{2} \div 1\frac{5}{7}$

Ex. $\frac{3}{16} + \frac{5}{16}$

Ex. $8\frac{4}{5} - 7\frac{4}{9}$

Ex. $\frac{1}{8} \cdot 7\frac{1}{2}$

1.2

$$3^2 = 3 \bullet 3 = 9 \text{ "3 squared"}$$

The 3 is the base, the 2 is the exponent or power.

Ex. 4^3

Ex. $\left(\frac{2}{3}\right)^5$

Order of Operations

Parentheses and Exponents

Multiplication and Division

Addition and Subtraction

PE Please Excuse

MD My Dear

AS Aunt Sally

Ex. $6[2 + 8(3^3)]$

Ex. $\frac{6(5+1) - 9(1+1)}{5(8-6) - 2^3}$

$>$ greater than

\geq greater than or equal to

$<$ less than

\leq less than or equal to

$=$ equal to

\neq not equal to

Ex. True or False?

$$6[2 + 3(2 + 5)] \leq 135$$

Ex. Write in symbols: Twelve is equal to twenty minus eight.

Ex. Write in words and decide whether true: $9 < 10$

Ex. Write the statement with the inequality symbol reversed while keeping the same meaning. $8 > 4$

1.3

Algebraic expression: A collection of numbers, variables, operations, and grouping symbols. There are no equal signs in expressions.

Equation: Statement that two algebraic expressions are equal. Equations must have an equal sign.

Ex. Find the value if $x=4$ and then if $x=6$:

$$5x^2 \qquad \frac{4x-1}{3x}$$

Ex. Find the value if $x=2$ and $y=1$ and then if $x=1$ and $y=5$:

$$4x + 2y + 7$$

Ex. Write each word phrase as an algebraic expression, using x as the variable.

Nine times a number

A number subtracted from fourteen

Ex. Decide whether the given number is a solution of the equation.

$$3r+5=8; 1$$

$$6a+2(a+3)=14; 2$$

Ex. Write each word sentence as an equation, using x as the variable. Find all solutions from the set $\{2, 4, 6, 8, 10\}$: The sum of six-fifths of a number and 2 is 14.

1.4

Number line:



Negative numbers on this side

0

Positive numbers on this side

Set-builder notation:

{ start with a brace

x , or some other variable, is introduced

| is usually read as “such that” or “so that”. Think also “as long as” or “provided that”.

x 's particular property is described

} end with another brace

{ x | x has some property }

The additive inverse is the number with the opposite sign. The absolute value is the distance from 0, so it will always be a positive value.

$$-(-x) = x$$

Ex. Write the opposite of the number and then the absolute value of the number.

$$-8$$

$$11$$

Ex. Identify the lesser of the two numbers.

$$-9, -14$$

$$|-8|, |-9|$$

Ex. True or False?

$$-8 > (-2)$$

$$|-12| \leq |-20|$$

$$-|-12| \leq -|-15|$$

Ex. Which is greater? 1.8 or -3.1 ?

1.5 The definition of subtraction: $a - b = a + (-b)$

Ex. Find each sum or difference.

$$12 + (-9)$$

$$-11 + (-4)$$

$$-12 + 10$$

$$8 - 13$$

$$-4-(5-12)$$

$$|-2-7|-|9-(-3)|$$

$$-3+5+(-12)$$

$$7-(-14)$$

1.6

Multiplying or dividing two numbers with the same sign gives a positive number.

Multiplying or dividing two numbers with opposite signs gives a negative number.

Ex. Find each product or quotient.

$$(-4)(-6)$$

$$(-8)(5)$$

$$(-12)(0)$$

$$\left(\frac{-5}{4}\right)\left(\frac{-6}{25}\right)$$

Ex. Find all integer factors of 36.

Ex. Find all integer factors of 17.

Ex. Evaluate; simplify completely.

$$(5-12)(19-4)$$

$$4(-8) + |4 - 15|$$

Ex. Evaluate; simplify completely.

$$\frac{-5(2) + [3(-2) - 4]}{-3 - (-1)}$$

Ex. Evaluate the expression if $x=6$, $y=-4$, and $a=3$.

$$5x - 4a^2$$

Ex. Write a numerical expression for each phrase and simplify: The sum of -18 and -6, divided by the product of 2 and -4.

Ex. Write in symbols, using x as the variable, then guess or use trial and error to find the solution from the set of integers between -12 and 12, inclusive: Four times a number is -36.

Ex. Find the average of 18, 12, 0, -4, and -10.